

# SEIR Model for Covid-19 Coronavirus

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## 1 Introduction

One of the most used models in the study of epidemics is the so-called SIR model that was proposed in [7] by W. O. Kermarck and A. G. McKendrick in 1927. As an extension of this model we have the SEIR model [5, 6], which we will use for this report.

Given a population of fixed size  $N$  in which an epidemic has been triggered that spreads by contagion in a time  $t$  (measured in days) individuals can be in three different states:

- Susceptible  $S(t)$ : number of individuals who can contract the disease.
- Infected  $I(t)$ : number of infected persons who infect  $\beta$  individuals each day.
- Recovered  $R(t)$ : number of individuals who have overcome the disease or have died

By considering  $t_I$  as the time in which an individual is in the infected phase, i.e., assuming that the recovery rate is  $\gamma := 1/t_I$ , and being  $t_0$  the study start time and  $t_0 + T$  the end time, we have the SIR model:

$$\begin{cases} S'(t) = -\beta I(t)S(t)/N, & t_0 < t \leq t_0 + T, \\ I'(t) = \beta I(t)S(t)/N - \gamma I(t), \\ R'(t) = \gamma I(t), \\ S(t_0) = S_0, \quad I(t_0) = I_0, \quad R(t_0) = R_0, \end{cases}$$

where  $S_0$ ,  $I_0$  and  $R_0$  are the initial data at time  $t_0$ . The origin of this model is motivated by the logistic behavior of the infected [7]. If we assume  $\gamma=0$ , that is, individuals never recover from the disease, we obtain the logistic model:

$$I' = \beta I \frac{S}{N} = \beta I \frac{N - I}{N} = \beta I \left(1 - \frac{I}{N}\right).$$

However, if  $\gamma > 0$ , those infected can recover and therefore stop infecting.

Once the SIR model is presented, we observe that it does not adapt well to the behavior of the coronavirus epidemic since this disease has an incubation period. As a solution, we introduce a new state:

- Exposed  $E(t)$ : number of individuals who have been infected but cannot infect,

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and we assume that an individual is for a time  $t_E$  in this state. This brings us to the SEIR model:

$$\begin{cases} S'(t) = -\beta I(t)S(t)/N, & t_0 < t \leq t_0 + T, \\ E'(t) = \beta I(t)S(t)/N - \sigma E(t), \\ I'(t) = \sigma E(t) - \gamma I(t), \\ R'(t) = \gamma I(t), \\ S(t_0) = S_0, \quad E(t_0) = E_0, \quad I(t_0) = I_0, \quad R(t_0) = R_0, \end{cases} \quad (1.1)$$

where the parameter  $\sigma := 1/t_E$  is the incubation rate of the disease and  $S_0, E_0, I_0$  and  $R_0$  are the initial data at time  $t_0$ . Also, another very important parameter in epidemics is the basic reproduction rate [2, 6], denoted by  $R_0$ . This rate represents the number of new infections produced by a single infected person during its infection stage, i.e.,

$$R_0 = \frac{\beta}{\gamma} = \beta t_I.$$

One of the most recent studies on the SEIR model applied to this disease is that conducted by José Manuel Gutiérrez and Juan Luis Varona [4]. They have located several works on the disease and mention that the “valid” parameters are

$$t_E = 7, \quad t_I = 5, \quad \text{y} \quad 2.68 \leq R_0 \leq 8.4. \quad (1.2)$$

On the other hand, another important point to consider in the model is the confinement produced on March 15, 2020. Due to containment measures (protection and isolation), the parameter  $\beta$  may change over time. For this purpose, we introduced a new parameter  $\alpha \in [0, 1]$  that will represent the containment. This will appear by multiplying  $\beta$  on the day  $t_\alpha$  that these measures are applied. Therefore, if it is only applied once, the new basic rate of reproduction for  $t \in [t_\alpha, T]$  would be

$$R_0(\alpha) := \beta_\alpha t_I \quad \text{donde} \quad \beta_\alpha := \alpha\beta.$$

The aim of this paper is to compare the SEIR model with the available real data on the spread of Covid-19 coronavirus in Spain. Specifically, we will use the number of deaths as a study variable:

- Deceased  $F(t)$ : number of individuals who have not overcome the disease

which is related to the number of recoveries by means of the mortality rate,  $\tau$ , in the sense

$$F(t) := \tau R(t).$$

We believe that this variable may be more significant than the number of people infected, but we have the problem of not knowing the mortality rate. The actual data can be downloaded in [10]. Specifically, we used the number of deaths between March 8 and April 3 (both included), i.e.,

$$t_0 = \text{March 7th, 2020} \quad \text{and} \quad t_\alpha = \text{April 3rd, 2020} = t_0 + 27. \quad (1.3)$$

The great disadvantage of this type of work is the lack of knowledge of a large number of parameters. Even setting the parameters of (1.2) and (1.3), we do not know  $\alpha, \tau$  and the initial data. Therefore, we propose the study by varying the parameters in a certain range and we see that values minimize the following weighted relative error

$$\text{Error} = \frac{1}{378} \sum_{t=t_0+1}^{t_0+27} \left| \frac{F(t) - F_t}{F(t)} \right| (t - t_0), \quad (1.4)$$

where  $F_t$  is the actual data of deceased for day  $t$  and  $378 = \sum_{t=1}^{27} t$ . We think that the latter values are more significant than the first, so we've decided to take the error into account. Later studies can be considered with different weights.

In the following section 2, we give the simulations obtained for Spain, the 17 Autonomous Communities, Ceuta and Melilla. Given an initial number of infected  $I_0$ , we consider

$$E_0 = \beta t_E I_0, \quad R_0 = 0, \quad S_0 = N - I_0 - E_0 - R_0, \quad \text{and} \quad T = 100,$$

and (1.3). In addition, we assume that the rest of the parameters vary as follows:

- $t_I \in [2, 8]$  with step 1.
- $t_E + t_I \in [9, 15]$  with step 1.
- $R_0 \in [2, 9]$  with step 0.5.
- $\alpha \in (0, 1]$  with step 0.1.
- $\tau \in (0, 0.1]$  with step 0.01.

Each simulation is composed of two experiments:

- **Experiment 1:** We calculate which parameters minimize error (1.4). In addition, with these parameters, we provide a figure with the number of infected and deceased along with the actual data of the deceased.
- **Experiment 2:** We give a table considering different situations according to the error. We calculate the number of cases that have error  $< \varepsilon$  and we obtain  $F(100)$  for each case. Consequently, we get the minimum number of deceased, the maximum number of deceased and the average number of deceased for the set of parameters that have an error  $< \varepsilon$ .

In Section 3, we give a summary of Experiment 2 by adding the sum of the minimum, maximum and average number of deaths from all the communities in order to compare it with the results for Spain. Since all parameters are measured in days, we believe that it is convenient that the variation be discrete, so we will use numerical step  $h = 1$  to solve (1.1). Even so, ignoring the execution time, all the simulations are easy to adapt for  $h$  small. The code is compiled in Python 3.7 on a 16GB RAM computer with a runtime of approximately half an hour for each simulation. Finally, it should be noted that the data in this report is far beyond reality: [1] describes the fitting and rescaling done to the data given by the optimized SEIR model, so that adjust the values given by the predictor to the actual data.

## 2 Simulations

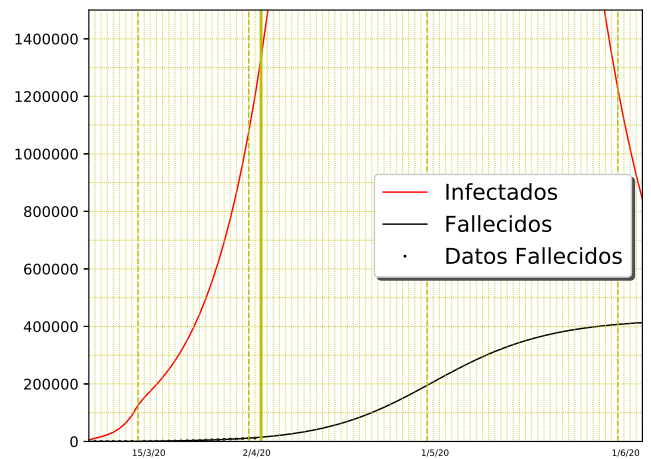
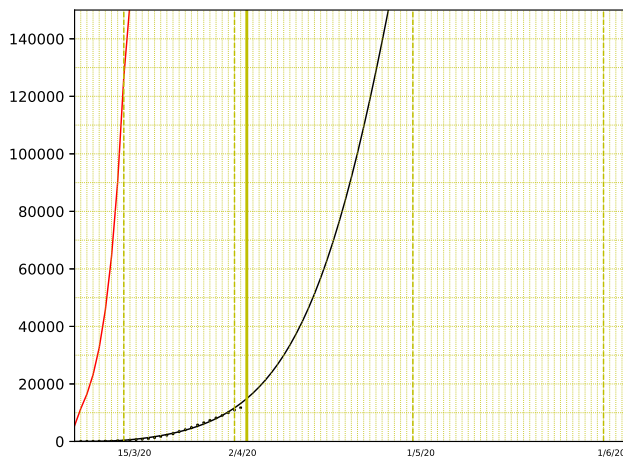
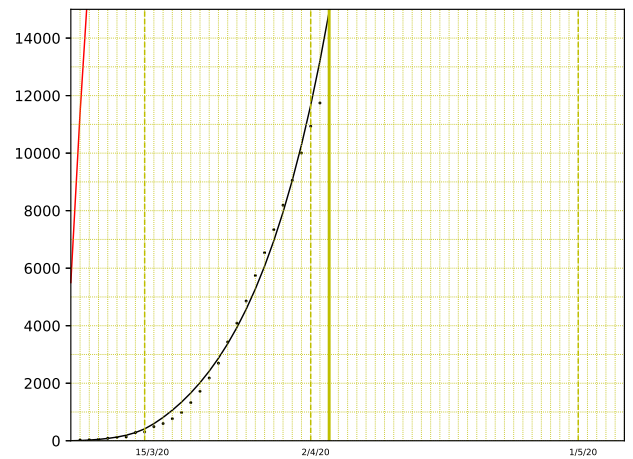
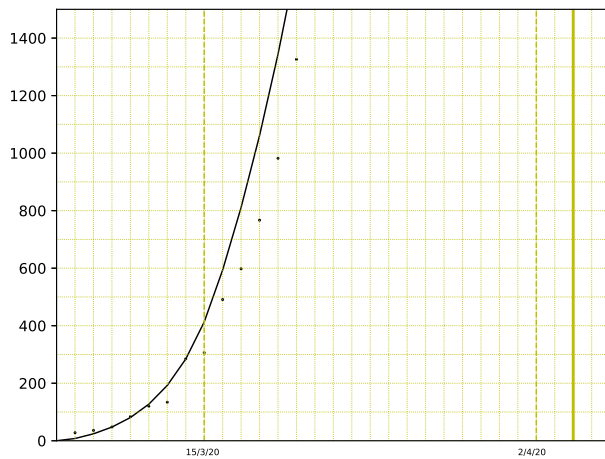
### 2.1 España

The number of people in Spain is  $N = 47,026,208$ . In addition, because the number of people infected on March 7 is 1006 (see [10]), we consider  $I_0 \in (0, 10000]$  with step 100.

#### Experiment 1

The minimum error achieved is 0.1 and the parameters that minimize it are

$$t_I = 7, \quad t_E = 3, \quad R_0 = 8.5, \quad I_0 = 5500, \quad \alpha = 0.3 \quad y \quad \tau = 0.01.$$



#### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	434293.0	23380.341970	4.699759e+06	1.466945e+06
0.25	263515.0	109679.548212	4.697640e+06	1.632180e+06
0.20	136872.0	237154.432831	4.690182e+06	1.723250e+06
0.15	28484.0	313586.172455	4.664876e+06	1.578013e+06

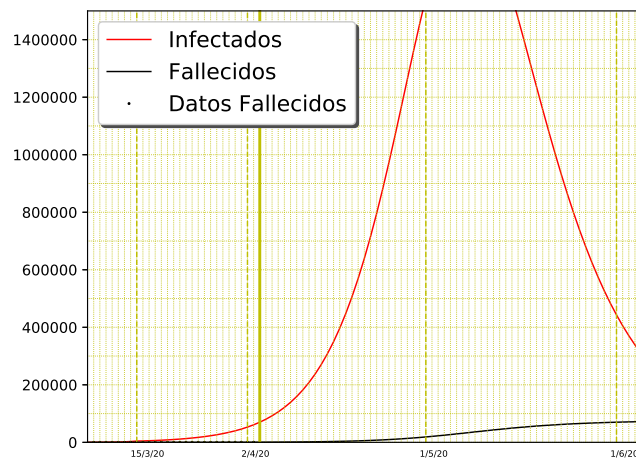
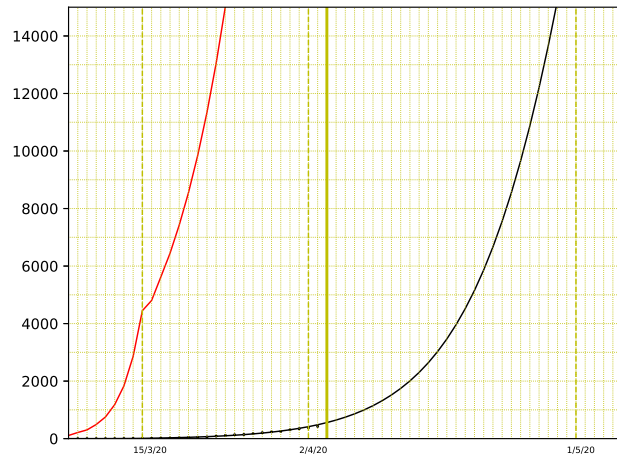
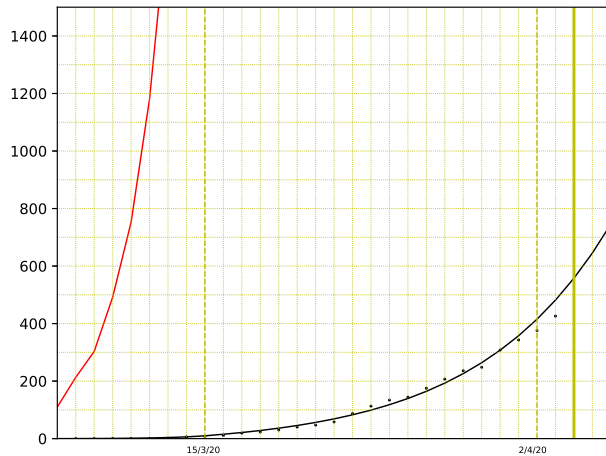
## 2.2 Andalusia (AN)

The number of people in Andalusia is  $N = 8,414,240$ . Also, because the number of people infected on March 7 is greater than 10 (see [10]), we consider  $I_0 \in (0, 1000]$  with step 10.

### Experiment 1

The minimum error achieved is 0.13 and the parameters that minimize it are

$$t_I = 8, \quad t_E = 1, \quad R_0 = 8.5, \quad I_0 = 110, \quad \alpha = 0.3 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	126155.0	8697.742976	840170.690223	205998.491268
0.25	62518.0	28425.143991	838465.875407	239898.364825
0.20	21671.0	60313.686073	831737.079923	259059.635871
0.15	96.0	71709.399133	717905.474130	247141.346888

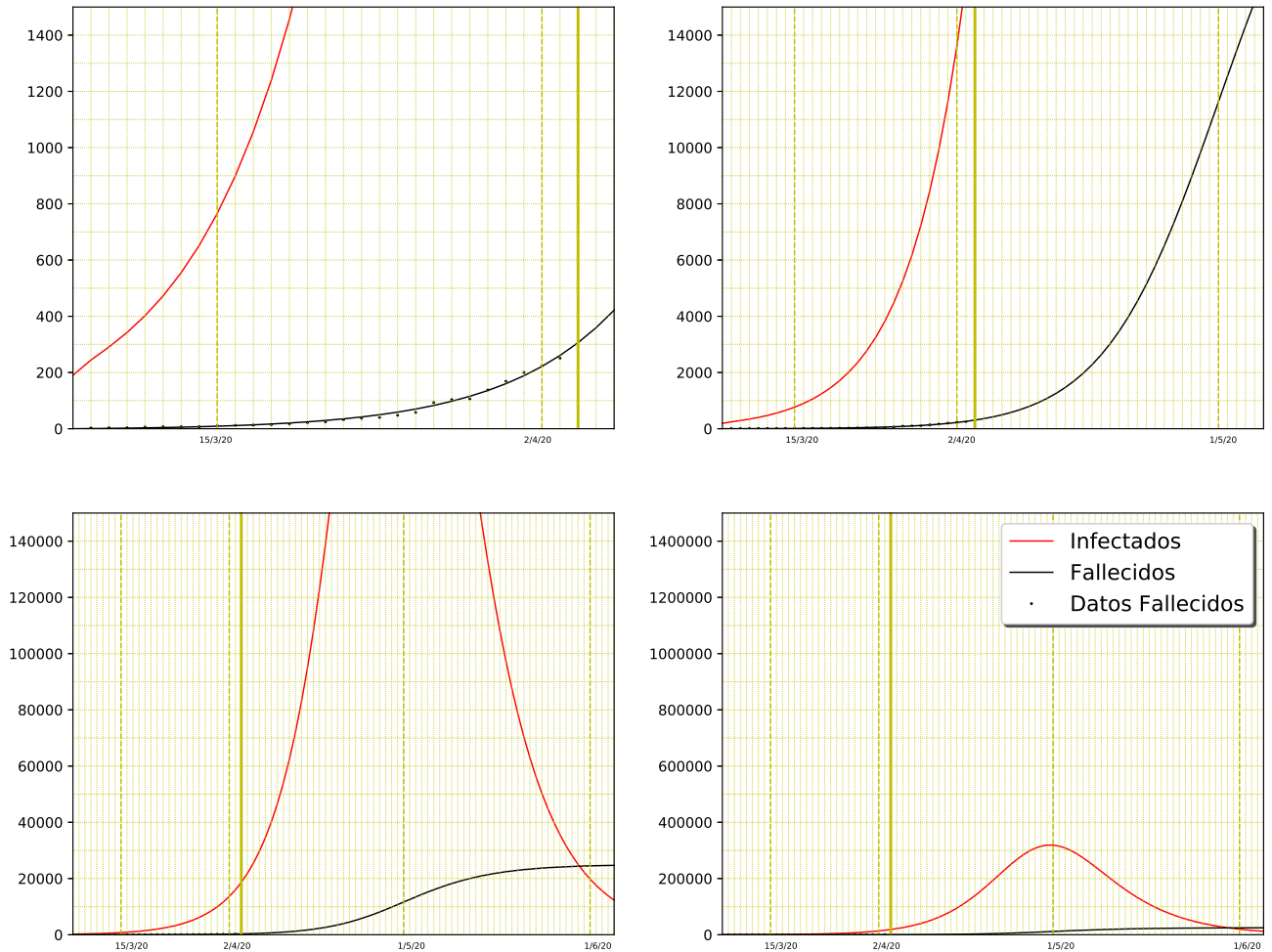
### 2.3 Aragón (AR)

The number of people in Aragón is  $N = 1,319,291$ . Also, because the number of people infected on March 7th is greater than 10 (see [10]), we consider  $I_0 \in (0, 1000]$  with step 10.

#### Experimento 1

The minimum error achieved is 0.14 and the parameters that minimize it are

$$t_I = 7, \quad t_E = 2, \quad R_0 = 3, \quad I_0 = 190, \quad \alpha = 1 \quad \text{y} \quad \tau = 0.02.$$



#### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	53580.0	7619.813950	131048.352504	37151.969222
0.25	20818.0	10394.325961	130865.385625	40928.131911
0.20	4693.0	11492.321922	130299.046365	43363.198998
0.15	42.0	12416.253457	127563.544538	50320.538417

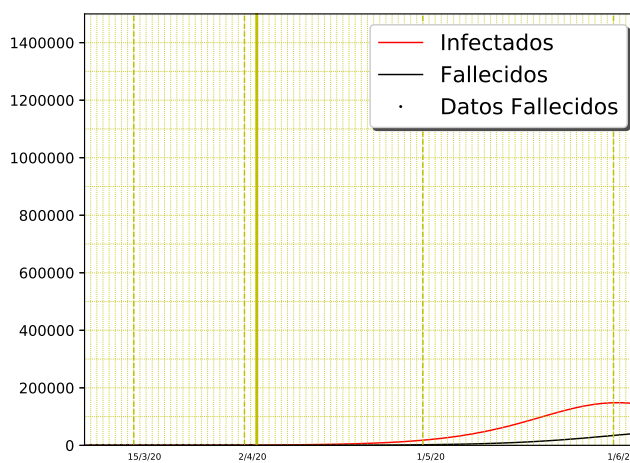
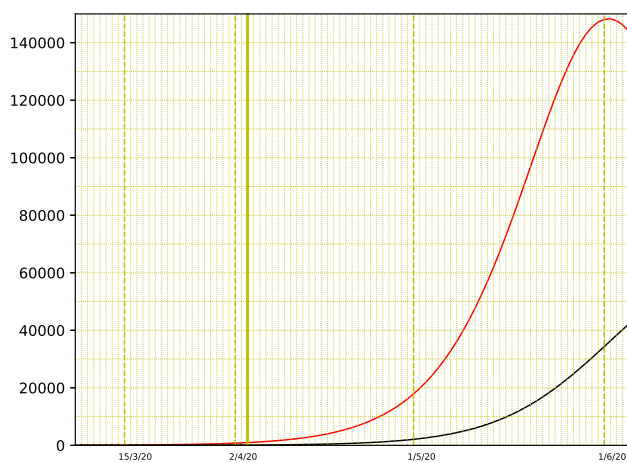
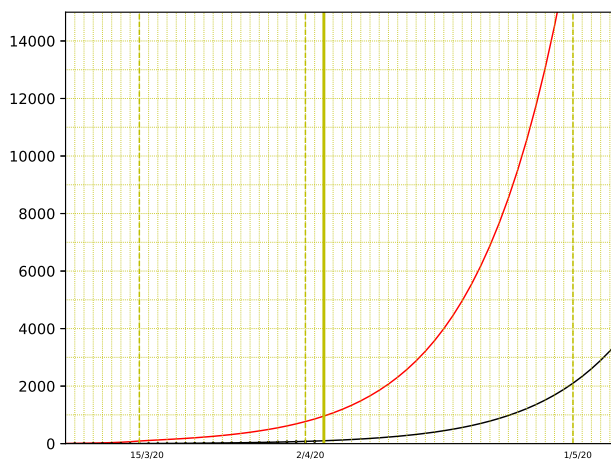
## 2.4 Asturias (AS)

The number of people in Asturias is  $N = 1,022,800$ . Also, because the number of people infected on March 7 is less than or equal to 10 (see [10]), we consider  $I_0 \in (0, 100]$  with step 1.

### Experiment 1

The minimum error achieved is 0.22 and the parameters that minimize it are

$$t_I = 6, \quad t_E = 5, \quad R_0 = 9, \quad I_0 = 4, \quad \alpha = 0.3 \quad \text{y} \quad \tau = 0.08.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	127171.0	1635.275411	101799.857766	28918.698727
0.25	12747.0	5463.566054	98256.309350	28154.753192

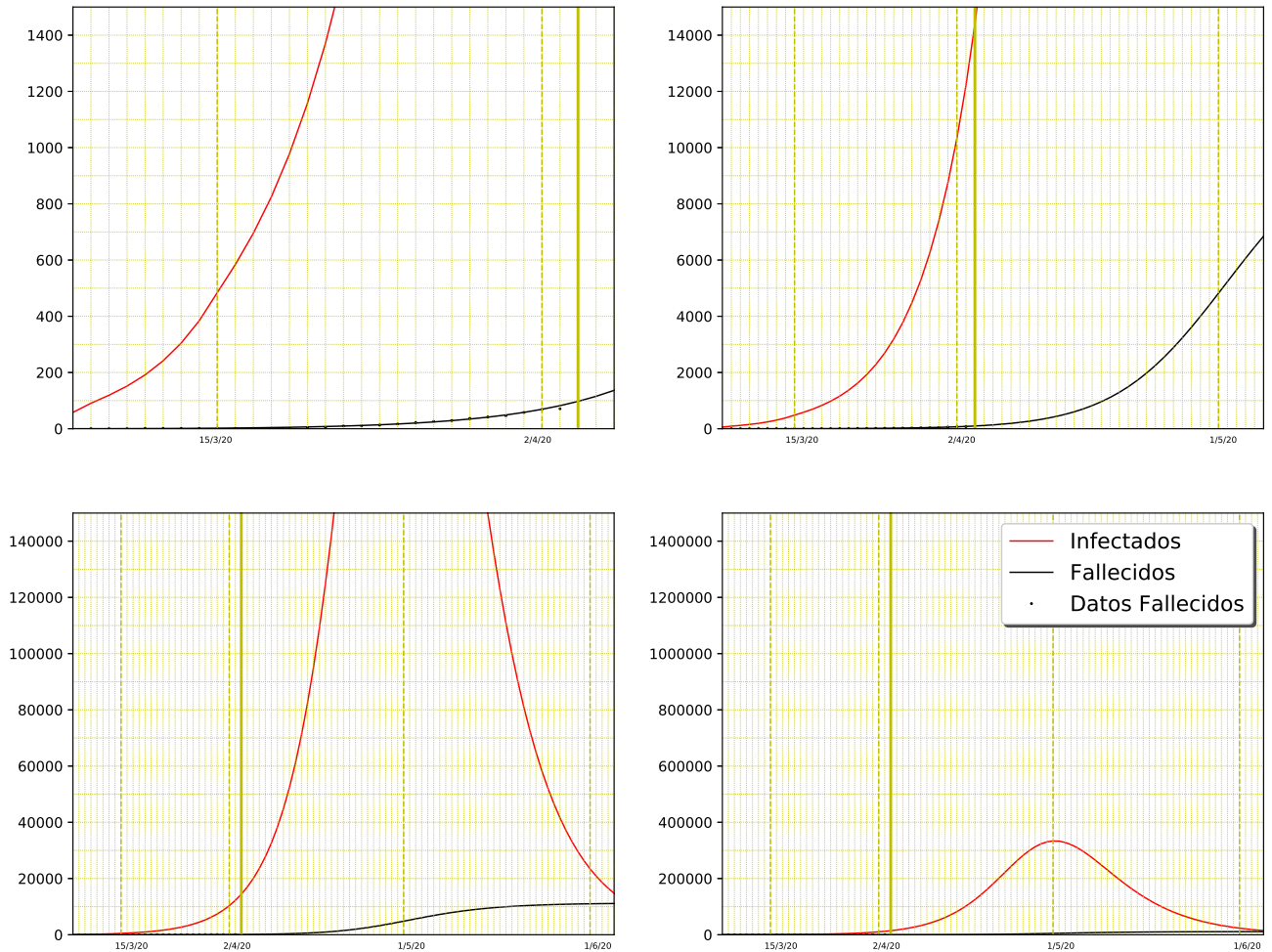
## 2.5 Balearic Islands (IB)

The number of people in the Balearic Islands is  $N = 1,149,460$ . In addition, because the number of people infected on March 7th is less than or equal to 10 (see [10]), we consider  $I_0 \in (0, 100]$  with step 1.

### Experiment 1

The minimum error achieved is 0.18 and the parameters that minimize it are

$$t_I = 8, \quad t_E = 3, \quad R_0 = 5.5, \quad I_0 = 58, \quad \alpha = 0.7 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	91968.0	7157.206935	114766.213655	36436.605511
0.25	39463.0	9812.421177	114529.258068	37857.088004
0.20	5751.0	10467.055522	113519.573515	37809.560100



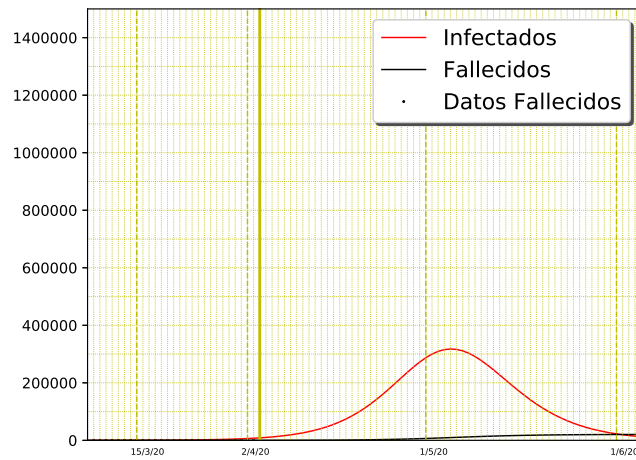
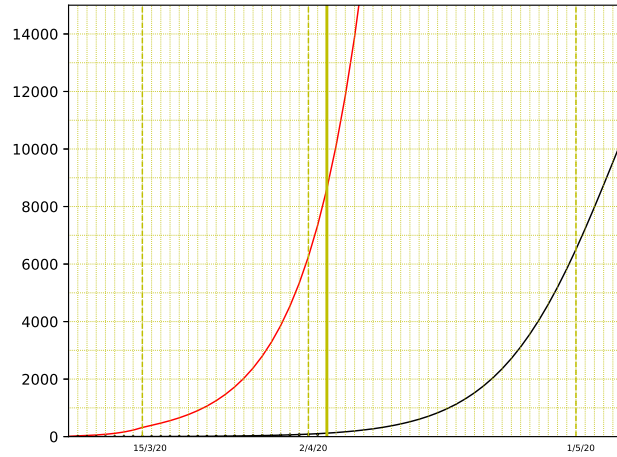
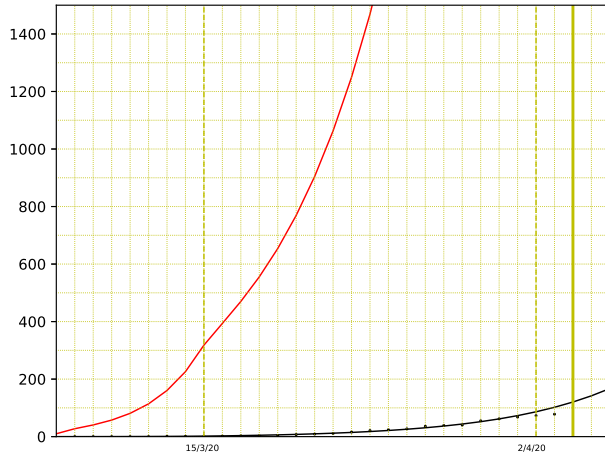
## 2.6 Canarias (CN)

The number of people in the Canary Islands is  $N = 2,153,389$ . Also, because the number of people infected on March 7th is greater than 10 (see [10]), we consider  $I_0 \in (0, 1000]$  with step 10.

### Experiment 1

The minimum error achieved is 0.17 and the parameters that minimize it are

$$t_I = 4, \quad t_E = 5, \quad R_0 = 8, \quad I_0 = 10, \quad \alpha = 0.4 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	18756.0	3895.516367	212170.971529	48662.755045
0.25	7990.0	9820.186170	210222.830374	56397.514640
0.20	856.0	18102.931081	196762.977341	50302.947495

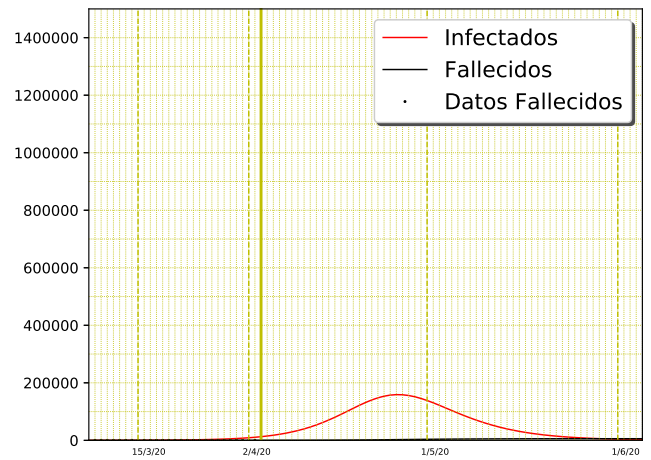
## 2.7 Cantabria (CB)

The number of people in Cantabria is  $N = 581,078$ . Also, because the number of infected on March 7th is greater than 10 (see [10]), we consider  $I_0 \in (0, 1000]$  with step 10.

### Experiment 1

The minimum error achieved is 0.28 and the parameters that minimize it are

$$t_I = 7, \quad t_E = 3, \quad R_0 = 7.5, \quad I_0 = 20, \quad \alpha = 0.5 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.3	582.0	5351.197224	54105.156388	12184.181757

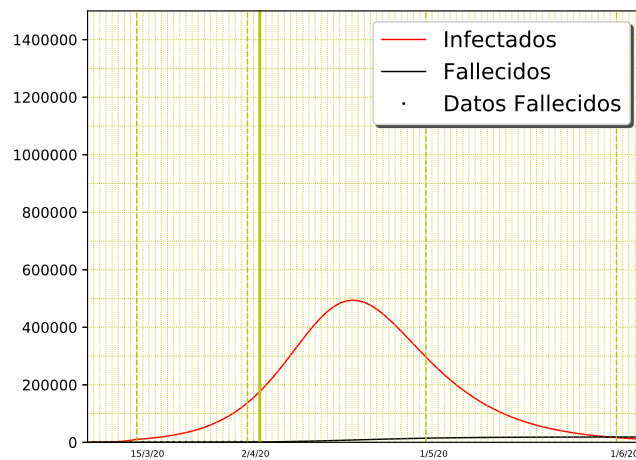
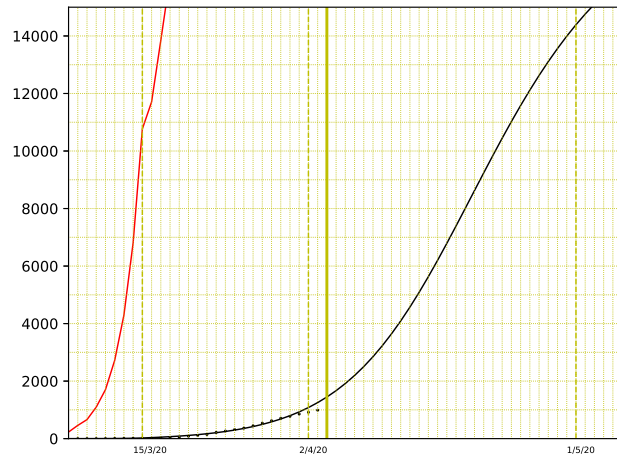
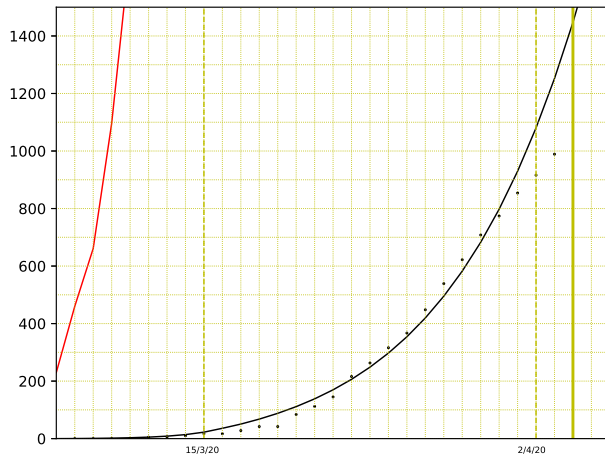
## 2.8 Castilla-La Mancha (CM)

The number of people in Castilla-La Mancha is  $N = 2,032,863$ . Also, because the number of people infected on March 7th is greater than 10 (see [10]), we consider  $I_0 \in (0, 1000]$  with step 10.

### Experiment 1

El error mínimo conseguido es 0.16 y los parámetros que lo minimizan son:

$$t_I = 8, \quad t_E = 1, \quad R_0 = 9, \quad I_0 = 203, \quad \alpha = 0.3 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	200597.0	11254.789683	203228.859230	72782.578466
0.25	82643.0	14261.576073	203115.596860	71500.863583
0.20	1502.0	17323.007447	197494.204531	56198.676479

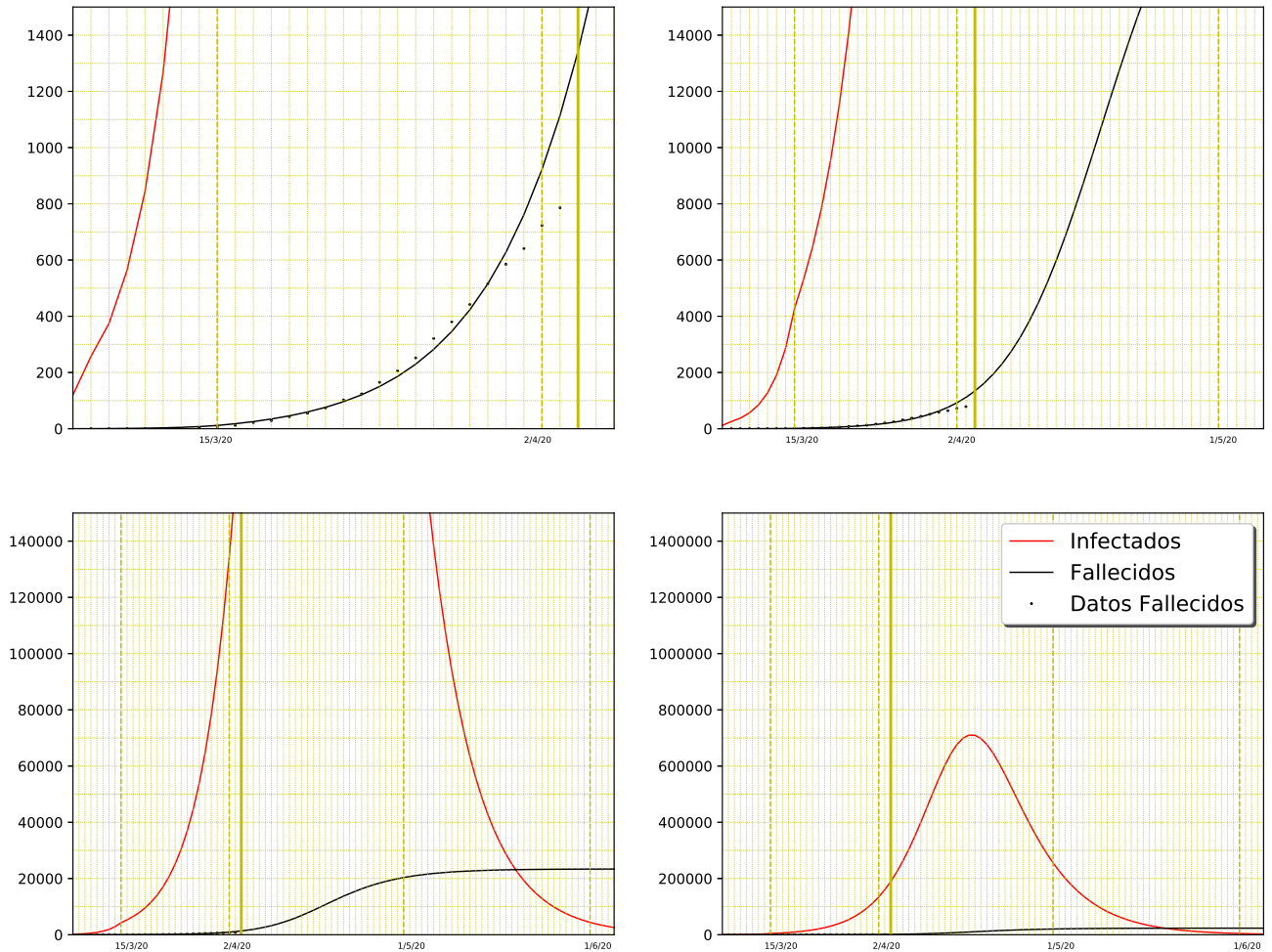
## 2.9 Castilla y León (CL)

The number of people in Castilla y Len is  $N = 2,399,548$ . Furthermore, because the number of people infected on March 7 is greater than 10 (see [10]), we consider  $I_0 \in (0, 1000]$  with step 10.

### Experiment 1

The minimum error achieved is 0.17 and the parameters that minimize it are

$$t_I = 7, \quad t_E = 2, \quad R_0 = 9, \quad I_0 = 120, \quad \alpha = 0.4 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	109010.0	18044.044311	239879.651375	82205.080716
0.25	45876.0	20330.537459	239755.723494	80862.371580
0.20	1111.0	22057.829388	238554.885481	65556.534085

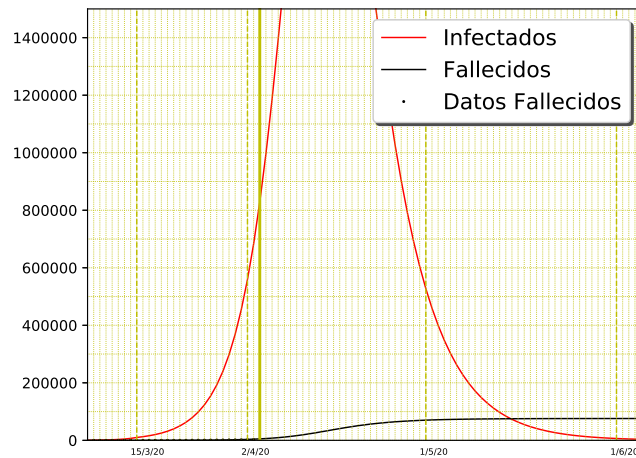
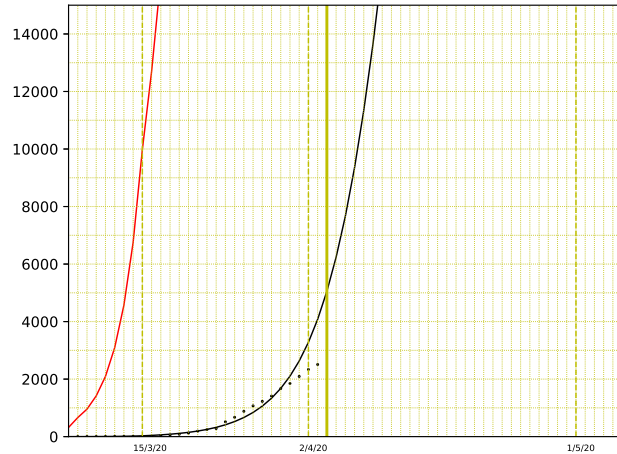
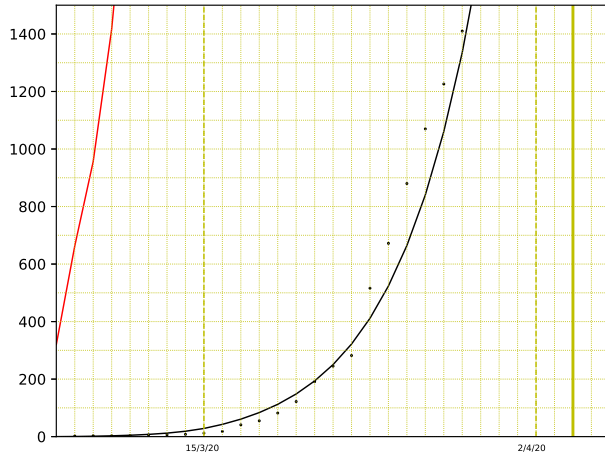
## 2.10 Cataluña (CT)

The number of people in Catalonia is  $N = 7,675,217$ . Also, because the number of people infected on March 7th is greater than 10 (see [10]), we consider  $I_0 \in (0, 1000]$  with step 10.

### Experiment 1

The minimum error achieved is 0.24 and the parameters that minimize it are

$$t_I = 7, \quad t_E = 2, \quad R_0 = 8.5, \quad I_0 = 320, \quad \alpha = 0.5 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	139598.0	62155.027679	767413.295613	312495.627480
0.25	581.0	69166.960755	761440.527253	202477.262942

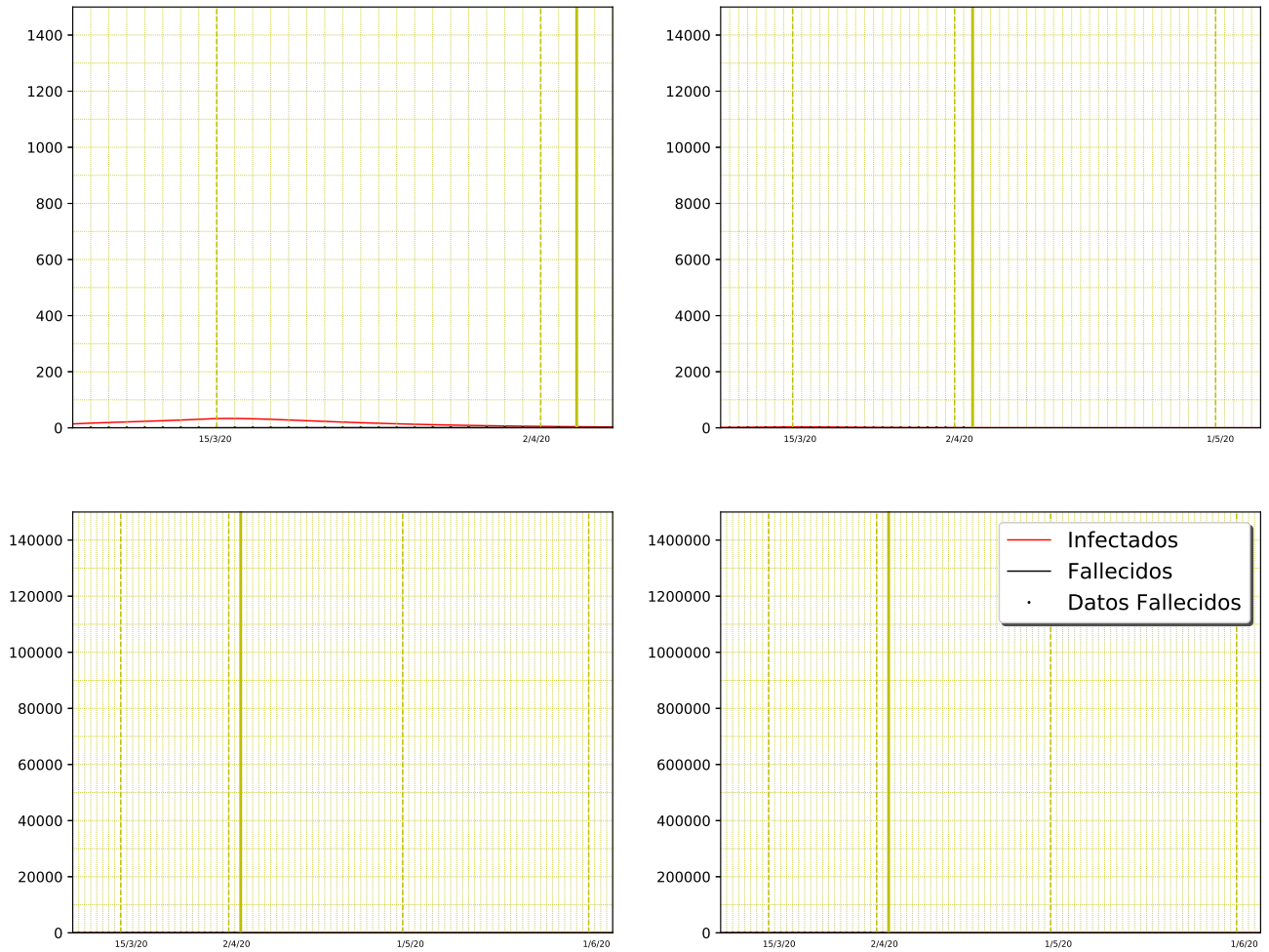
## 2.11 Ceuta (CE)

The number of people in Ceuta is  $N = 84,777$ . Also, because the number of people infected on March 7th is less than or equal to 10 (see [10]), we consider  $I_0 \in (0, 100]$  with step 1.

### Experiment 1

The minimum error achieved is 0.51 and the parameters that minimize it are

$$t_I = 5, \quad t_E = 4, \quad R_0 = 2, \quad I_0 = 14, \quad \alpha = 0.1 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

The errors that have been considered for this experiment are less than 0.3. As in this autonomous city there is no case with an error less than 0.3, we do not put the table.

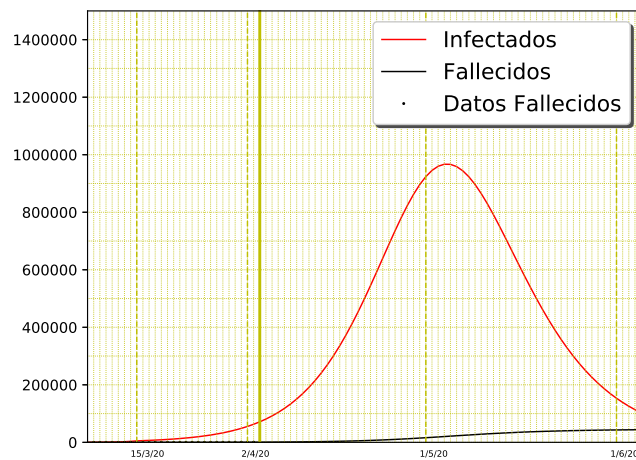
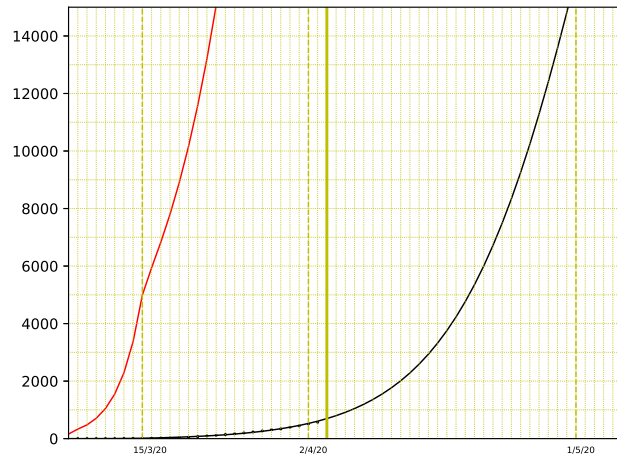
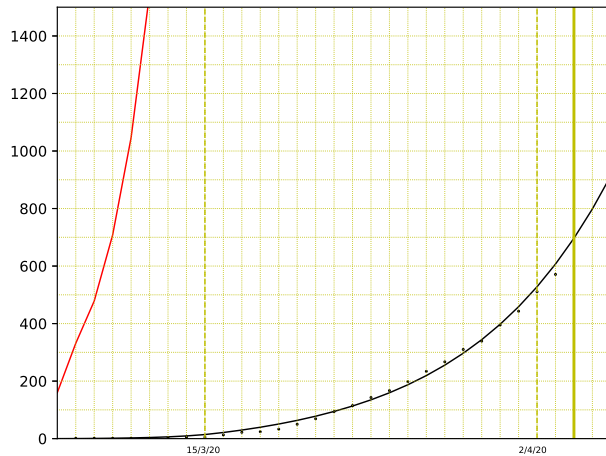
## 2.12 Valencian Community (VC)

The number of people in the Valencian Community is  $N = 5,003,769$ . Also, because the number of people infected on March 7th is greater than 10 (see [10]), we consider  $I_0 \in (0, 1000]$  with step 10.

### Experiment 1

The minimum error achieved is 0.13 and the parameters that minimize it are

$$t_I = 7, \quad t_E = 2, \quad R_0 = 8.5, \quad I_0 = 160, \quad \alpha = 0.3 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	157938.0	9408.595131	499816.644065	139982.192416
0.25	79043.0	23731.149510	498933.722495	155065.905229
0.20	25923.0	38743.260010	495688.503489	158986.132936
0.15	198.0	41556.639434	418695.957194	135558.591047

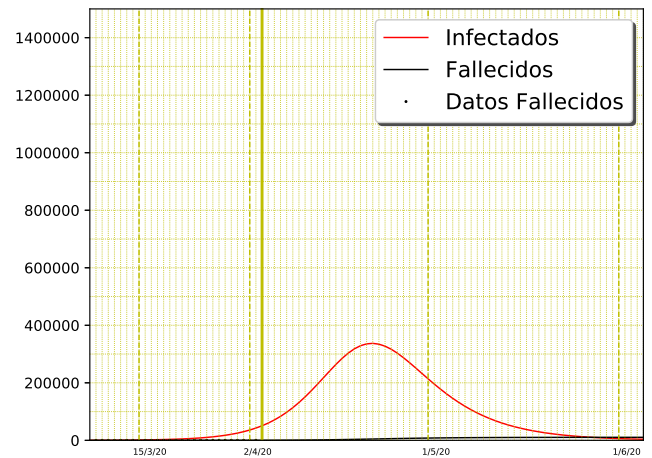
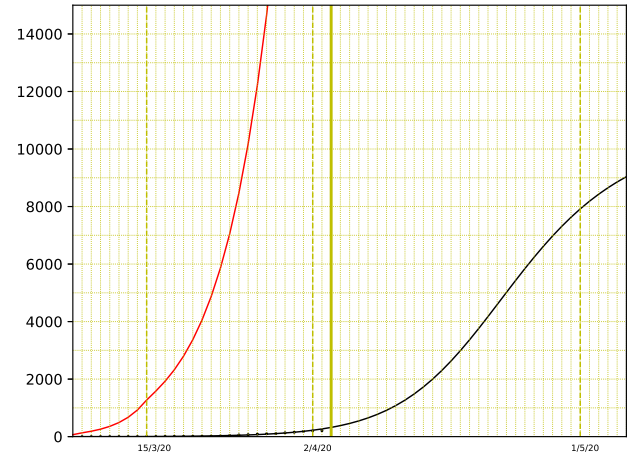
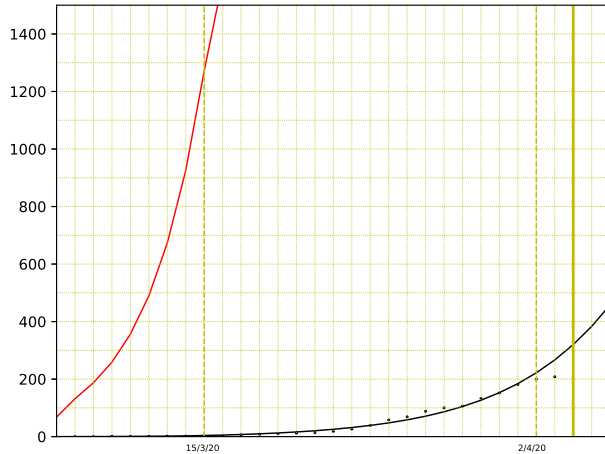
### 2.13 Extremadura (EX)

The number of people in Extremadura is  $N = 1,067,710$ . Also, because the number of people infected on March 7th is less than or equal to 10 (see [10]), we consider  $I_0 \in (0, 100]$  with step 1.

#### Experiment 1

The minimum error achieved is 0.17 and the parameters that minimize it are

$$t_I = 8, \quad t_E = 3, \quad R_0 = 8.5, \quad I_0 = 68, \quad \alpha = 0.5 \quad \text{y} \quad \tau = 0.01.$$



#### numerical Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	220026.0	6837.750954	106748.967387	42264.663761
0.25	107595.0	8964.609209	106722.465281	40368.696777
0.20	19406.0	9809.453340	106645.231282	36428.246488



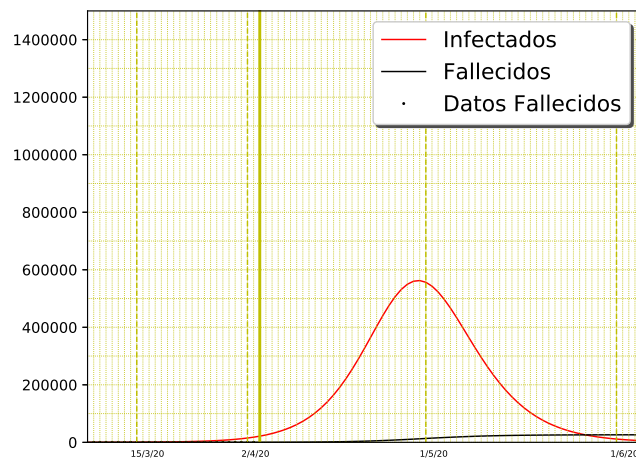
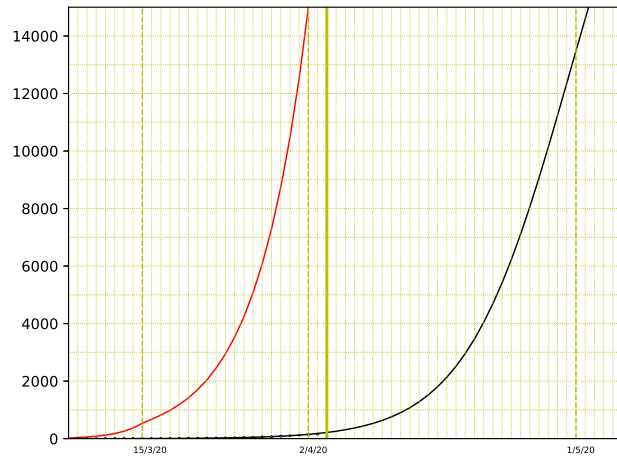
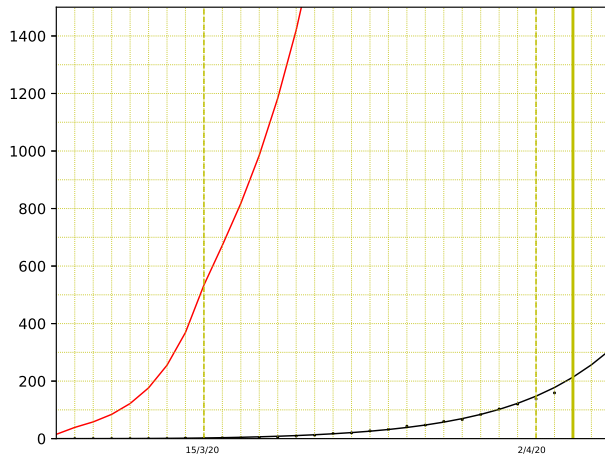
## 2.14 Galicia (GA)

The number of people in Galicia is  $N = 2,699,499$ . Furthermore, since the number of people infected on March 7th is less than or equal to 10 (see [10]), we consider  $I_0 \in (0, 100]$  with step 1.

### Experiment 1

The minimum error achieved is 0.14 and the parameters that minimize it are

$$t_I = 5, \quad t_E = 4, \quad R_0 = 9, \quad I_0 = 15, \quad \alpha = 0.4 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	141089.0	19774.341136	269806.223215	95851.119502
0.25	73332.0	22953.392751	269623.090641	95131.571635
0.20	27590.0	24543.457771	269281.480966	93121.965785
0.15	440.0	25411.294227	265834.482972	87903.435765

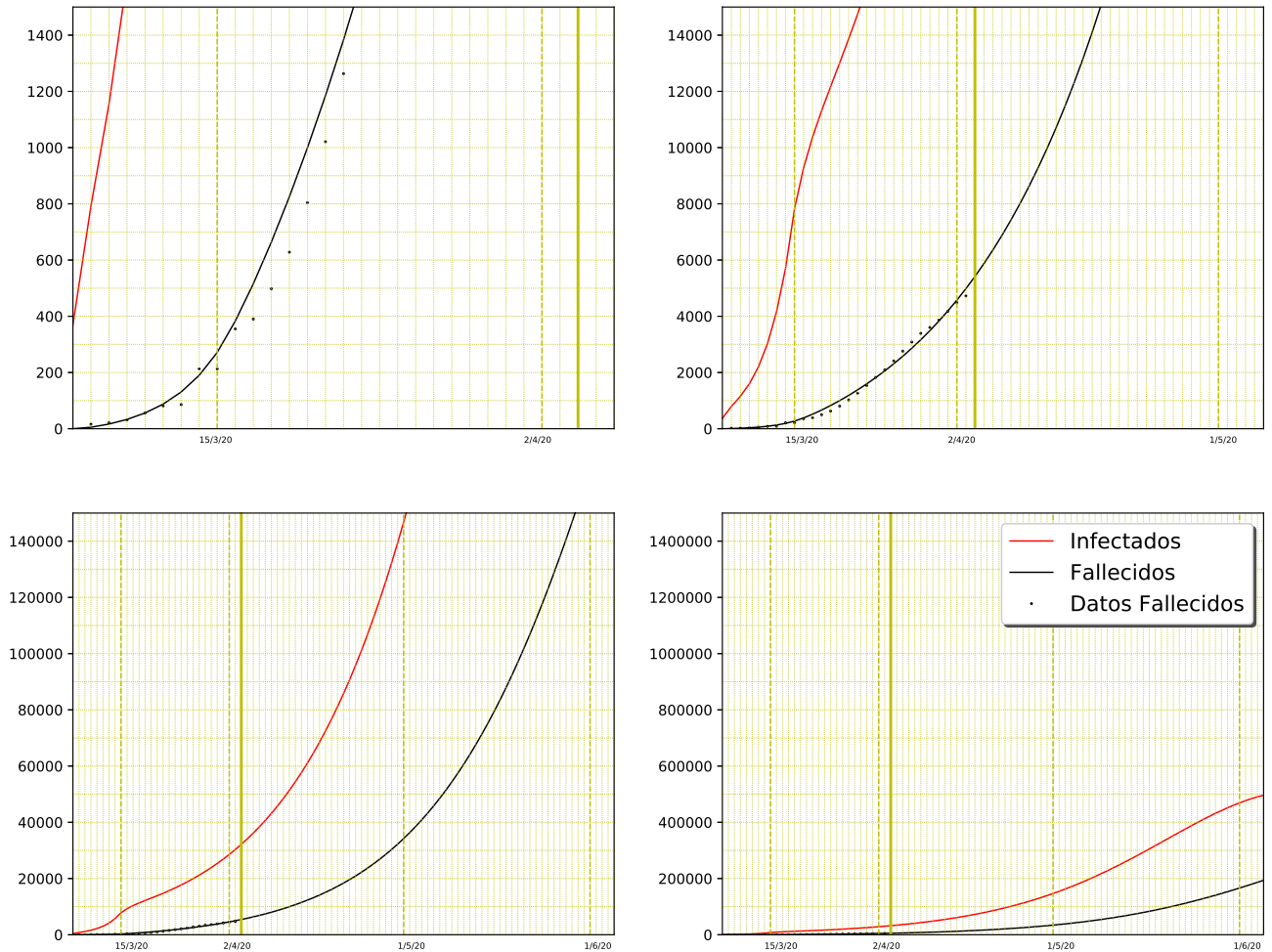
## 2.15 Community of Madrid (MD)

The number of people in the Community of Madrid is  $N = 6,663,394$ . In addition, because the number of people infected on March 7 is greater than 10 (see [10]), we consider  $I_0 \in (0, 1000]$  with step 10.

### Experiment 1

The minimum error achieved is 0.08 and the parameters that minimize it are

$$t_I = 7, \quad t_E = 4, \quad R_0 = 9, \quad I_0 = 370, \quad \alpha = 0.2 \quad \text{y} \quad \tau = 0.1.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	609420.0	4700.937723	665573.480714	296577.055242
0.25	417046.0	6285.347517	664642.572036	306055.061778
0.20	241954.0	9661.867909	662712.207194	323656.568153
0.15	95361.0	30219.019929	652122.478190	321964.806318
0.10	4958.0	41186.688549	549438.543185	267202.220104

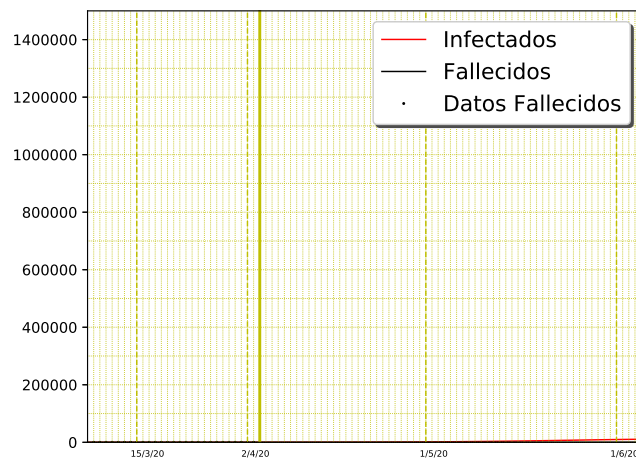
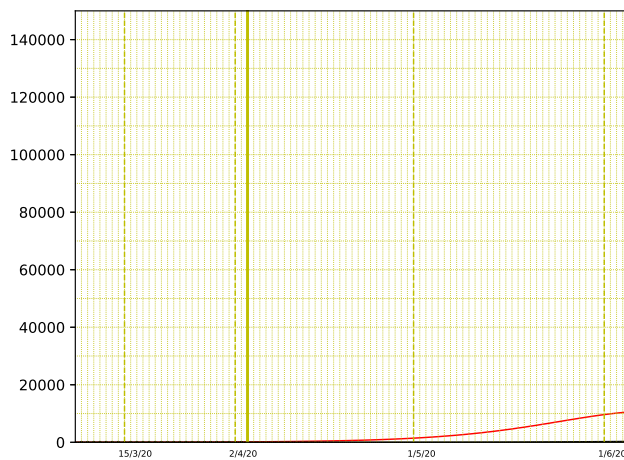
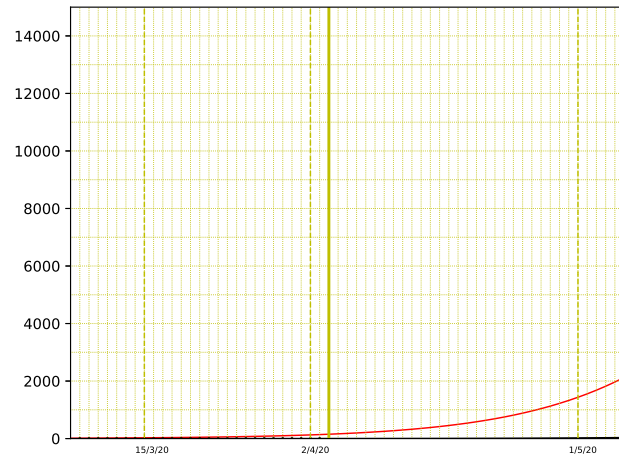
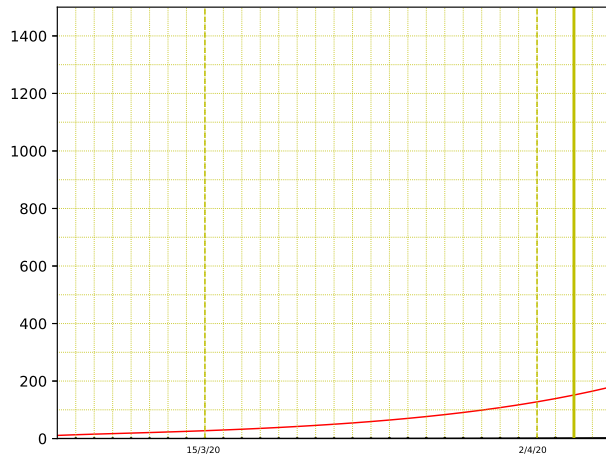
## 2.16 Melilla (ME)

The number of people in Melilla is  $N = 86,487$ . Also, because the number of people infected on March 7 is less than or equal to 10 (see [10]), we consider  $I_0 \in (0, 100]$  with step 1.

### Experiment 1

The minimum error achieved is 0.5 and the parameters that minimize it are

$$t_I = 7, \quad t_E = 6, \quad R_0 = 2.5, \quad I_0 = 11, \quad \alpha = 1 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

The errors that have been considered for this experiment are less than 0.3. As in this autonomous city there is no case with an error less than 0.3, we do not put the table.

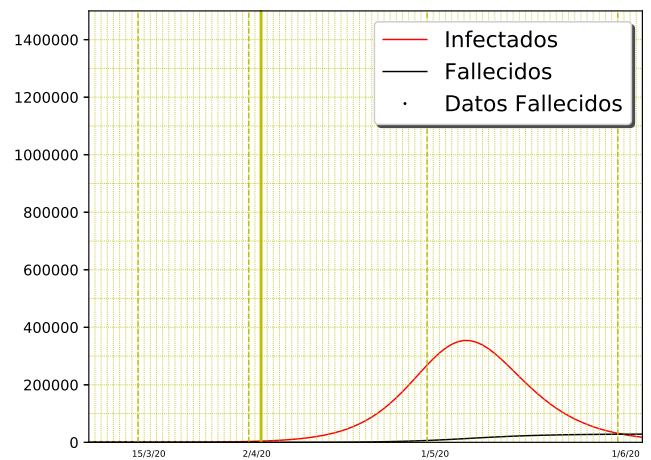
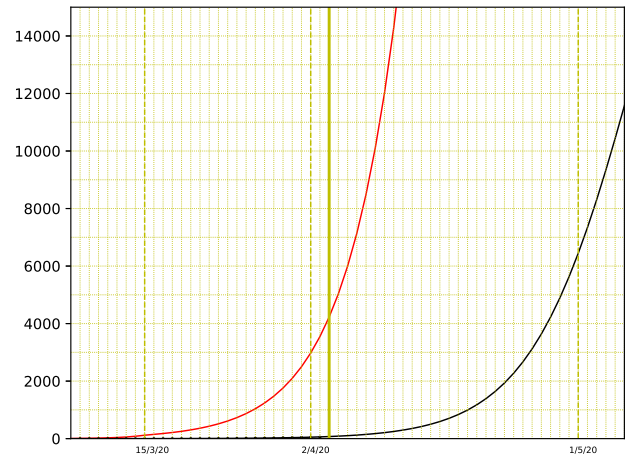
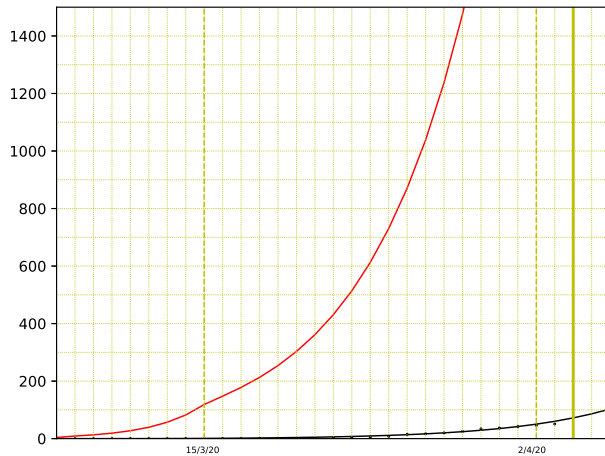
## 2.17 Región de Murcia (MC)

The number of people in the Region of Murcia is  $N = 1,493,898$ . Also, because the number of people infected on March 7 is less than or equal to 10 (see [10]), consideramos  $I_0 \in (0, 100]$  with step 1.

### Experiment 1

The minimum error achieved is 0.38 and the parameters that minimize it are

$$t_I = 6, \quad t_E = 3, \quad R_0 = 8.5, \quad I_0 = 4, \quad \alpha = 0.4 \quad \text{y} \quad \tau = 0.02.$$



### Experiment 2

The errors that have been considered for this experiment are less than 0.3. As in this community there is no case with error less than 0.3, we do not put the table.

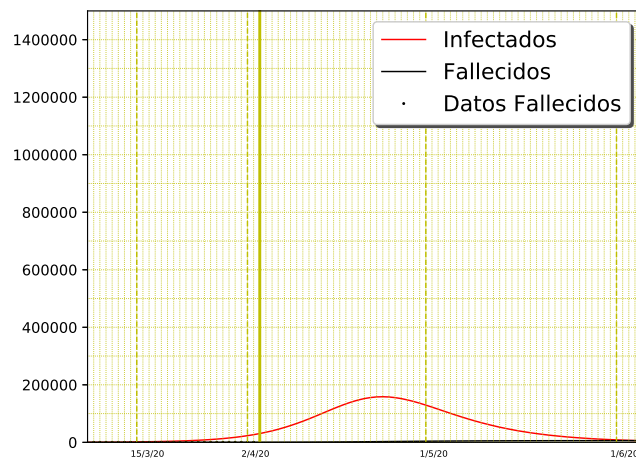
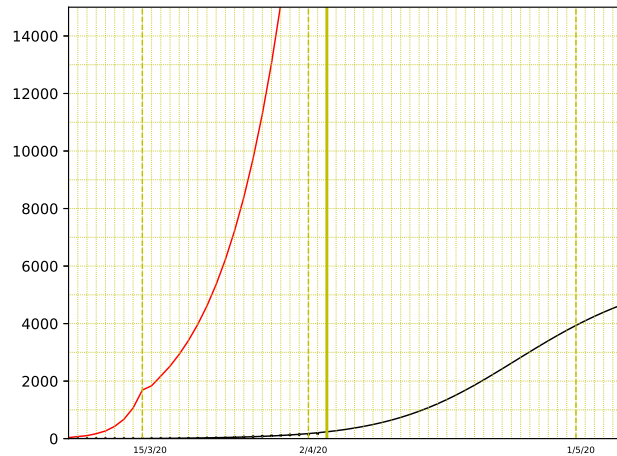
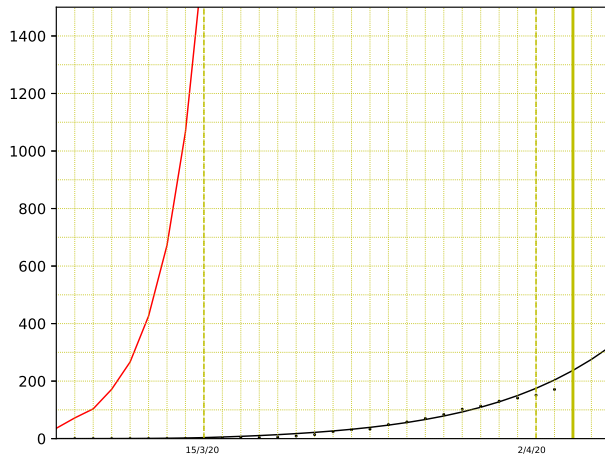
## 2.18 Comunidad Foral de Navarra (NC)

The number of people in Comunidad Foral de Navarra is  $N = 654.214$ . In addition, because the number of people infected on March 7 is less than or equal to 10 (see [10]), we consider  $I_0 \in (0, 100]$  with step 1.

### Experiment 1

El error mínimo conseguido es 0.24 y los parámetros que lo minimizan son:

$$t_I = 8, \quad t_E = 1, \quad R_0 = 9, \quad I_0 = 36, \quad \alpha = 0.3 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	108180.0	5246.253426	65336.178204	25839.239966
0.25	85.0	6010.233899	57191.584232	13338.631161

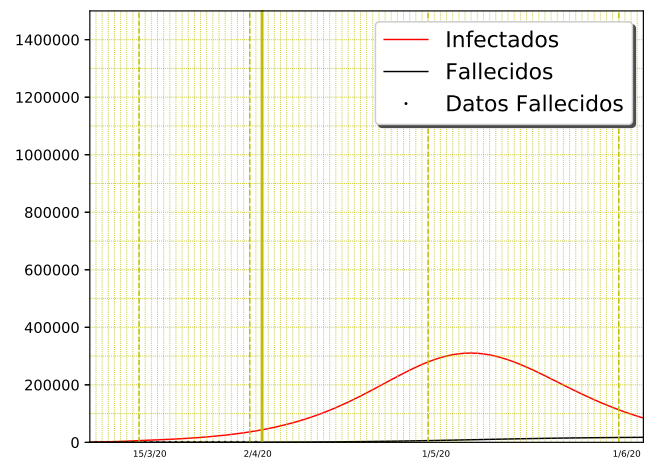
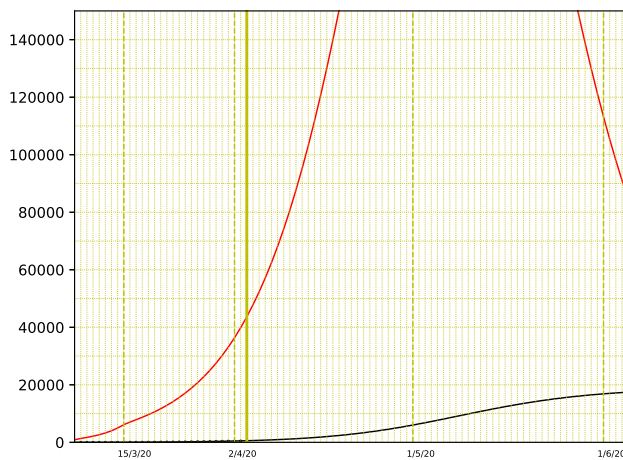
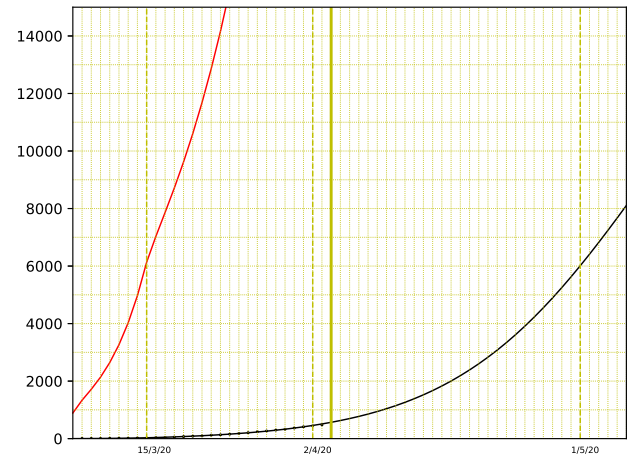
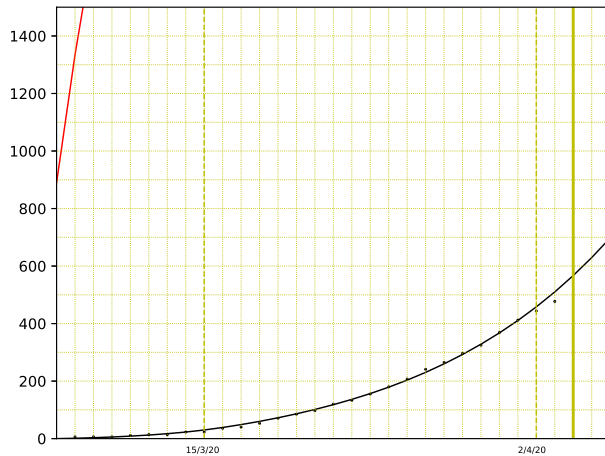
## 2.19 Basque Country (PV)

The number of people in the Basque Country is  $N = 2.207.776$ . Furthermore, because the number of people infected on March 7 is greater than 10 (see [10]), we consider  $I_0 \in (0, 1000]$  with step 10.

### Experiment 1

The minimum error achieved is 0.05 and the parameters that minimize it are:

$$t_I = 7, \quad t_E = 3, \quad R_0 = 4.5, \quad I_0 = 890, \quad \alpha = 0.5 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	434421.0	443.418020	219673.365049	36633.750400
0.25	270164.0	679.894560	218920.984439	42846.184672
0.20	148849.0	1395.300226	217546.135052	50789.267050
0.15	68001.0	5823.944919	214377.915963	58860.707753
0.10	18471.0	13523.257261	207470.590248	65216.859735

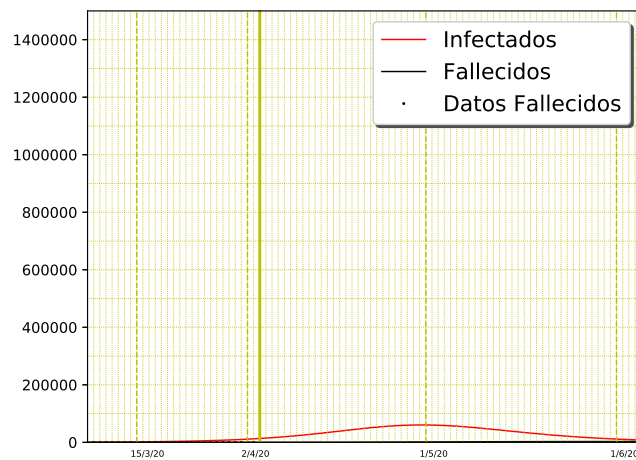
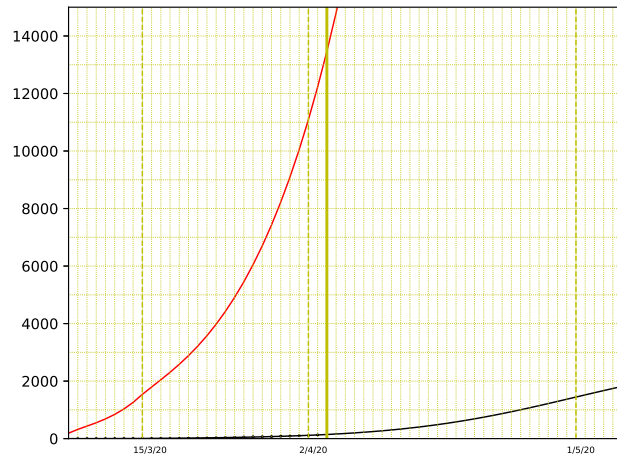
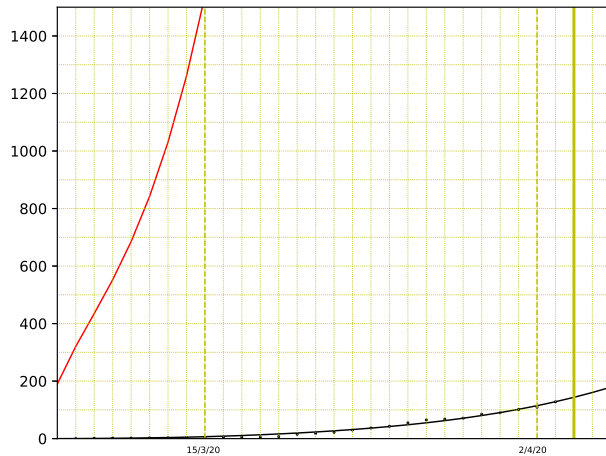
## 2.20 La Rioja (RI)

The number of people in La Rioja is  $N = 316.798$ . Also, because the number of people infected on March 7 is greater than 10 (see [10]), we consider  $I_0 \in (0, 1000]$  with step 10.

### Experiment 1

The minimum error achieved is 0.14 and the parameters that minimize it are:

$$t_I = 8, \quad t_E = 6, \quad R_0 = 6.5, \quad I_0 = 190, \quad \alpha = 0.5 \quad \text{y} \quad \tau = 0.01.$$



### Experiment 2

$\varepsilon$	N° Cases	Minimum Deceased	Maximum Deceased	Average number of deaths
0.30	80072.0	195.179695	31613.892299	6788.571962
0.25	39100.0	692.694787	31540.378533	8256.281911
0.20	13573.0	1882.648835	31301.865486	9228.835061
0.15	1.0	2992.970887	2992.970887	2992.970887

### 3 Spain vs Autonomous Communities

Finally, we give a global table of Experiment 2 for the values of  $\varepsilon = 0.3, 0.25, 0.2$  and  $0.15$ . In addition, we add a new row with the sum of the autonomous communities and cities (Communities) with which it is possible to compare with the row for Spain. Since there are some autonomous communities and cities that do not have all the values of  $\varepsilon$ , we will calculate  $F(100)$  with the values of Experiment 1 and we will choose this value as the minimum, maximum and average number of deaths in the case that you do not have  $\varepsilon$ .

The table for errors less than 0.3 is:

Name	Minimum Deceased	Maximum Deceased	Average number of deaths
Spain	23380.341970	4.699759e+06	1.466945e+06
AN	8697.742976	8.401707e+05	2.059985e+05
AR	7619.813950	1.310484e+05	3.715197e+04
AS	1635.275411	1.017999e+05	2.891870e+04
IB	7157.206935	1.147662e+05	3.643661e+04
CN	3895.516367	2.121710e+05	4.866276e+04
CB	5351.197224	5.410516e+04	1.218418e+04
CM	11254.789683	2.032289e+05	7.278258e+04
CL	18044.044311	2.398797e+05	8.220508e+04
CT	62155.027679	7.674133e+05	3.124956e+05
CE	1.117182	1.117182e+00	1.117182e+00
VC	9408.595131	4.998166e+05	1.399822e+05
EX	6837.750954	1.067490e+05	4.226466e+04
GA	19774.341136	2.698062e+05	9.585112e+04
MD	4700.937723	6.655735e+05	2.965771e+05
ME	446.752080	4.467521e+02	4.467521e+02
MC	28827.820037	2.882782e+04	2.882782e+04
NC	5246.253426	6.533618e+04	2.583924e+04
PV	443.418020	2.196734e+05	3.663375e+04
RI	195.179695	3.161389e+04	6.788572e+03
Communities	201692.779919	4.552427e+06	1.510048e+06



The table for errors less than 0.25 is:

Name	Minimum Deceased	Maximum Deceased	Average number of deaths
Spain	109679.548212	4.697640e+06	1.632180e+06
AN	28425.143991	8.384659e+05	2.398984e+05
AR	10394.325961	1.308654e+05	4.092813e+04
AS	5463.566054	9.825631e+04	2.815475e+04
IB	9812.421177	1.145293e+05	3.785709e+04
CN	9820.186170	2.102228e+05	5.639751e+04
CB	5681.988003	5.681988e+03	5.681988e+03
CM	14261.576073	2.031156e+05	7.150086e+04
CL	20330.537459	2.397557e+05	8.086237e+04
CT	69166.960755	7.614405e+05	2.024773e+05
CE	1.117182	1.117182e+00	1.117182e+00
VC	23731.149510	4.989337e+05	1.550659e+05
EX	8964.609209	1.067225e+05	4.036870e+04
GA	22953.392751	2.696231e+05	9.513157e+04
MD	6285.347517	6.646426e+05	3.060551e+05
ME	446.752080	4.467521e+02	4.467521e+02
MC	28827.820037	2.882782e+04	2.882782e+04
NC	6010.233899	5.719158e+04	1.333863e+04
PV	679.894560	2.189210e+05	4.284618e+04
RI	692.694787	3.154038e+04	8.256282e+03
Communities	271949.717175	4.479184e+06	1.454096e+06

The table for errors less than 0.2 is:

Name	Minimum Deceased	Maximum Deceased	Average number of deaths
Spain	237154.432831	4.690182e+06	1.723250e+06
AN	60313.686073	8.317371e+05	2.590596e+05
AR	11492.321922	1.302990e+05	4.336320e+04
AS	58887.033107	5.888703e+04	5.888703e+04
IB	10467.055522	1.135196e+05	3.780956e+04
CN	18102.931081	1.967630e+05	5.030295e+04
CB	5681.988003	5.681988e+03	5.681988e+03
CM	17323.007447	1.974942e+05	5.619868e+04
CL	22057.829388	2.385549e+05	6.555653e+04
CT	75913.881557	7.591388e+04	7.591388e+04
CE	1.117182	1.117182e+00	1.117182e+00
VC	38743.260010	4.956885e+05	1.589861e+05
EX	9809.453340	1.066452e+05	3.642825e+04
GA	24543.457771	2.692815e+05	9.312197e+04
MD	9661.867909	6.627122e+05	3.236566e+05
ME	446.752080	4.467521e+02	4.467521e+02
MC	28827.820037	2.882782e+04	2.882782e+04
NC	6011.392677	6.011393e+03	6.011393e+03
PV	1395.300226	2.175461e+05	5.078927e+04
RI	1882.648835	3.130187e+04	9.228835e+03
Communities	401562.804166	3.667313e+06	1.360272e+06

The table for errors less than 0.15 is:

Name	Minimum Deceased	Maximum Deceased	Average number of deaths
Spain	313586.172455	4.664876e+06	1.578013e+06
AN	71709.399133	7.179055e+05	2.471413e+05
AR	12416.253457	1.275635e+05	5.032054e+04
AS	58887.033107	5.888703e+04	5.888703e+04
IB	11225.251730	1.122525e+04	1.122525e+04
CN	20604.428111	2.060443e+04	2.060443e+04
CB	5681.988003	5.681988e+03	5.681988e+03
CM	18717.501826	1.871750e+04	1.871750e+04
CL	23397.434790	2.339743e+04	2.339743e+04
CT	75913.881557	7.591388e+04	7.591388e+04
CE	1.117182	1.117182e+00	1.117182e+00
VC	41556.639434	4.186960e+05	1.355586e+05
EX	10541.372763	1.054137e+04	1.054137e+04
GA	25411.294227	2.658345e+05	8.790344e+04
MD	30219.019929	6.521225e+05	3.219648e+05
ME	446.752080	4.467521e+02	4.467521e+02
MC	28827.820037	2.882782e+04	2.882782e+04
NC	6011.392677	6.011393e+03	6.011393e+03
PV	5823.944919	2.143779e+05	5.886071e+04
RI	2992.970887	2.992971e+03	2.992971e+03
Communities	450385.495847	2.659749e+06	1.164998e+06

## References

- [1] ALEJA, D., CRIADO, R., & ROMANCE, M. ,“Predicting the evolution of the Covid-19’ epidemic using a SEIR Model” (2020).
- [2] DIEKMANN, O., HEESTERBEEK, H., & BRITTON, T. ,“Mathematical tools for understanding infectious disease dynamics” (Vol. 7). Princeton University Press (2012).
- [3] GANDHI, K.R.R., & CASELLA, F. , “Non-Pharmaceutical Interventions (NPIs) to Reduce COVID-19 Mortality”. Available at SSRN 3560688 (2020).
- [4] GUTIÉRREZ, J.M. & VARONA, J.L., “Análisis de la posible evolución de la epidemia de coronavirus COVID-19 por medio de un modelo SEIR”, (in spanish) available at <https://belenus.unirioja.es/jvarona/coronavirus/SEIR-coronavirus.pdf> (2020).
- [5] HETHCOTE, H. W., “The mathematics of infectious diseases”, *SIAM Review*, 42(4), 599-653 (2000).
- [6] KEELING, M. J., & ROHANI, P. , “Modeling infectious diseases in humans and animals”. Princeton University Press (2011).
- [7] KERMACK, W. O., & MCKENDRICK, A. G., “A contribution to the mathematical theory of epidemics”. *Proceedings of the Royal Society of London. Series A*, 115(772), 700-721 (1927).
- [8] KUCHARSKI, A. J., RUSSELL, ET. AL., “Early dynamics of transmission and control of COVID-19: a mathematical modelling study”. *The Lancet Infectious Diseases* (2020).
- [9] LIU, Y., GAYLE, A. A., WILDER-SMITH, A., & ROCKLOV, J., “The reproductive number of COVID-19 is higher compared to SARS coronavirus”. *Journal of travel medicine* (2020).
- [10] [10] Official data from the Spanish Government on the spread of Covid-19 provided by the Carlos III Health Institute (ISCIII), updated daily at [https://covid19.isciii.es/recursos/serie\\_historica\\_acumulados.csv](https://covid19.isciii.es/recursos/serie_historica_acumulados.csv).